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# Almost Contra-Pre-Open and Almost Contra-Pre-Closed Mappings

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## Abstract

The aim of this paper is to introduce and study the concepts of almost contra-pre-open and almost contra-pre-closed mappings.

**Keywords:** Open set, Closed set, Open map, Closed map, Contra-Pre-Open Map, Contra-Pre-Closed Map, Almost Contra-Pre-Open Map and Almost Contra-Pre-Closed Map.

**AMS Classification:** 54C10, 54C08, 54C05.

## Introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open and closed mappings are one such mappings which are studied for different types of open and closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced -open and -closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced -open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of open and closed maps called almost contra-pre-open and almost contra-pre-closed maps. Throughout the paper X, Y means topological spaces $(X, \tau)$ and $(Y,\sigma)$ on which no separation axioms are assured.

## Preliminaries

**Definition 2.1:** $A\subseteq X$ is said to be

a) Regular open[pre-open; semi-open; α-open; β-open] if $A = int(cl(A))$ [$A \subseteq int(cl(A))$; $A\subseteq cl(int(A))$; $A\subseteq int(cl(int(A)))$; $A\subseteq cl(int(cl(A)))$] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = cl(int(A))$[$cl(int(A)) \subseteq A$; $int(cl(A))\subseteq A$; $cl(int(cl(A)))\subseteq A$; $int(cl(int(A)))\subseteq A$]

b) g-closed[rg-closed] if $cl(A)\subset U$[$rcl(A)\subset U$] whenever $A\subset U$ and U is open[r-open] in X and g-open[rg-open] if its complement X - A is g-closed[rg-closed].

**Definition 2.2:** A function $f:X\to Y$ is said to be

a) continuous[resp: semi-continuous, r-continuous] if the inverse image of every open set is open [resp: semi open, regular open] and g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].
b) irresolute [resp: r-irresolute] if the inverse image of every semi open [resp: regular open] set is semi open. [resp: regular open].
c) open[resp: semi-open; pre-open] if the image of every open set in $X$ is open[resp: semi-open; pre-open] in $Y$.
d) closed[resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].
e) contra-open[resp: contra semi-open; contra pre-open] if the image of every open set in $X$ is closed[resp: semi-closed; pre-closed] in $Y$.
f) contra closed[resp: contra semi-closed; contra pre-closed] if the image of every closed set in $X$ is open[resp: semi-open; pre-open] in $Y$.

g) **Remark 1:** We have the following implication diagrams for open sets and closed sets.

semi-open $\rightarrow\ \rightarrow$ $\beta$-open semi-closed $\rightarrow\ \rightarrow$ $\beta$-closed
$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$
r-open $\rightarrow$ open $\rightarrow$ $\alpha$-open $\rightarrow$ pre-open r-closed $\rightarrow$ closed $\rightarrow$ $\alpha$-closed $\rightarrow$ pre-closed

**Definition 2.3:** *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

## Almost Contra Pre-Open Mappings

**Definition 3.1:** A function $f: X \rightarrow Y$ is said to be almost contra pre-open if the image of every r-open set in *X* is pre-closed in *Y*.

**Example 1:** Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, X\}$; $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f:X \rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then *f* is almost contra-pre-open and contra-pre-open.

**Example 2:** Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Assume $f: X \rightarrow Y$ be the identity map. Then *f* is not almost contra-pre-open.

**Note 1:** We have the following implication diagram among the open maps.

c.s.o. $\rightarrow\ \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$ c.$\beta$.o
$\downarrow$ $\downarrow$
al.c.s.o. $\rightarrow\ \rightarrow\ \rightarrow \rightarrow \rightarrow$ al.c.$\beta$.o
$\uparrow$ $\uparrow$
Al.c.r.o $\rightarrow \rightarrow \rightarrow$ al.c.o. $\rightarrow$ al.c.$\alpha$.o. $\rightarrow$ al.c.p.o.
$\uparrow$ $\uparrow$ $\uparrow$
c.r.o $\rightarrow \rightarrow \rightarrow \rightarrow$ c.o. $\rightarrow \rightarrow$ c.$\alpha$.o. $\rightarrow \rightarrow$ c.p.o. None is reversible.

**Theorem 3.1:** Every contra-pre-open map is almost contra-pre-open but not conversely.
**Proof:** Let $A \subseteq X$ be r-open $\Rightarrow$ A is open $\Rightarrow$ $f(A)$ is pre-closed in *Y*. since $f:X \rightarrow Y$ is contra-pre-open. Hence *f* is almost contra-pre-open.

**Example 3:** Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, X\}$; $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f:X \rightarrow Y$ be defined $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then *f* is almost contra-pre-open but not contra-pre-open.

**Theorem 3.2:**
(i) If $RC(Y) = PC(Y)$ then *f* is almost contra r-open iff *f* is almost contra pre-open.
(ii) If $PC(Y) = RC(Y)$ then *f* is almost contra-open iff *f* is almost contra pre-open.
(iii) If $PC(Y) = \alpha C(Y)$ then *f* is almost contra $\alpha$-open iff *f* is almost contra pre-open.
**Proof:** Follows from note 1 and remark 1.

**Theorem 3.3:** If *f* is open[r-open] and *g* is contra pre-open then $g\ o\ f$ is almost contra pre-open.
**Proof:** Let $A \subseteq X$ be r-open $\Rightarrow f(A)$ is open[r-open] in $Y \Rightarrow g(f(A)) = g\ o\ f(A)$ is pre-closed in *Z*. Hence *gof* is almost contra pre-open.

**Corollary 3.1:** If *f* is open[r-open] and *g* is contra-open[contra *r*-open] then *gof* is almost contra pre-open.

**Theorem 3.4:** If *f* is almost contra open[almost contra-r-open] and *g* is pre-closed then $g\ o\ f$ is almost contra-pre-open.
**Proof:** Let $A \subseteq X$ be r-open in X $\Rightarrow f(A)$ is closed[r-closed] in Y $\Rightarrow g(f(A)) = g\bullet f(A)$ is pre-closed in Z. Hence $g\bullet f$ is almost contra pre-open.

**Theorem 3.5:** If *f* is almost contra open[almost contra-r-open] and *g* is *r*-closed then *gof* is almost contra-pre-open.
**Proof:** Let $A \subseteq X$ be r-open in X $\Rightarrow f(A)$ is closed in Y $\Rightarrow g(f(A))$ is *r*-closed in Z $\Rightarrow g(f(A)) = g\bullet f(A)$ is pre-closed in Z. Hence $g\bullet f$ is almost contra pre-open.

**Theorem 3.6:** If $f$: $X \to Y$ is almost contra pre-open then $p\overline{(f(A))} \subseteq f(\bar{A})$

**Proof:** Let $A \subseteq X$ and $f$: $X \to Y$ be almost contra pre-open. Then $f(\bar{A})$ is pre-closed in $Y$ and $f(A) \subseteq f(\bar{A})$. This implies $p\overline{(f(A))} \subseteq p\overline{(f(\bar{A}))}$ $\to$ (1)

Since $f(\bar{A})$ is pre-closed in $Y$, $p\overline{(f(\bar{A}))} = f(\bar{A})$ $\to$ (2)

Using (1) & (2) we have $p\overline{(f(A))} = f(\bar{A})$ for every subset A of $X$.

**Remark 2:** Converse is not true in general as shown by the following example.

**Example 4:** Let $X = Y = \{a, b, c\}$ $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f$: $X \to Y$ be the identity map. Then $p\overline{(f(A))} \subseteq f(\bar{A})$ for every subset A of $X$. But $f$ is not contra-pre-open since $f(\{a\}) = \{a\}$ is not pre-closed.

**Example 5:** Let $X = Y = \{a, b, c\}$ $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f$: $X \to Y$ be the identity map. Then $p\overline{(f(A))} \subseteq f(\bar{A})$ for every subset A of $X$. But $f$ is not almost contra-preopen since $f(\{a\}) = \{a\}$ and $f(\{b\}) = \{b\}$ are not pre-closed.

**Corollary 3.2:** If $f:X \to Y$ is almost contra $r$-open then $p\overline{(f(A))} \subseteq f(\bar{A})$.

**Theorem 3.7:** If $f:X \to Y$ is almost contra pre-open and $A \subseteq X$ is open, $f(A)$ is $\tau_{pre}$-closed in $Y$.

**Proof:** Let $A \subseteq X$ and $f:X \to Y$ be almost contra pre-open $\Rightarrow p\overline{(f(A))} \subseteq f(\bar{A})$ (by theorem 3.6.) $\Rightarrow p\overline{(f(A))} \subseteq f(A)$ since $f(A) = f(\bar{A})$ as A is open. But $f(A) \subseteq p\overline{(f(A))}$. Therefore we have $f(A) = p\overline{(f(A))}$. Hence $f(A)$ is $\tau_{pre}$-closed in $Y$.

**Corollary 3.3:** If $f:X \to Y$ is almost contra $r$-open, then $f(A)$ is $\tau_{pre}$-closed in $Y$ if A is $r$-open set in $X$.

**Theorem 3.8:** $f:X \to Y$ is almost contra pre-open iff for each subset S of $Y$ and each r-closed set U containing $f^{-1}(S)$, there is an pre-open set V of $Y$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

**Proof:** Assume $f:X \to Y$ is almost contra pre-open. Let $S \subseteq Y$ and U be an r-closed set of $X$ containing $f^{-1}(S)$. Then $X$-U is r-open in $X$ and $f(X\text{-}U)$ is pre-closed in $Y$ as $f$ is almost contra pre-open and $V = Y - f(X\text{-}U)$ is pre-open in $Y$. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y - f(X\text{-}U)) = f^{-1}(Y) - f^{-1}(f(X\text{-}U)) = f^{-1}(Y) - (X\text{-}U) = X - (X\text{-}U) = U$

Conversely Let F be r-open in $X \Rightarrow F^c$ is r-closed. Then $f^{-1}(f(F^c)) \subseteq F^c$.

By hypothesis there exists an pre-open set V of $Y$, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is pre-closed in $Y$. Therefore $f$ is almost contra pre-open.

**Remark 3:** Composition of two almost contra pre-open maps is not almost contra pre-open in general.

**Theorem 3.9:** Let $X$, $Y$, $Z$ be topological spaces and every pre-closed set is $r$-open in $Y$. Then the composition of two almost contra pre-open[almost contra $r$-open] maps is almost contra pre-open.

**Proof:** (a) Let $f:X \to Y$ and $g:Y \to Z$ be almost contra pre-open maps. Let A be any r-open set in $X \Rightarrow f(A)$ is pre-closed in $Y \Rightarrow f(A)$ is r-open in $Y$ (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is pre-closed in $Z$. Therefore $g \circ f$ is almost contra pre-open.

(b) Let $f:X \to Y$ and $g:Y \to Z$ be almost contra pre-open maps. Let A be any r-open set in $X \Rightarrow f(A)$ is $r$-closed in $Y \Rightarrow f(A)$ is pre-closed in $Y \Rightarrow f(A)$ is r-open in $Y$ (by assumption) $\Rightarrow g(f(A))$ is $r$-closed in $Z \Rightarrow gof(A)$ is pre-closed in $Z$. Therefore $gof$ is almost contra pre-open.

**Theorem 3.10:** Let $X$, $Y$, $Z$ be topological spaces and Y is discrete topological space in $Y$. Then the composition of two almost contra pre-open[almost contra $r$-open] maps is almost contra pre-open.

**Theorem 3.11:** If $f:X \to Y$ is g-open, $g:Y \to Z$ is contra pre-open and $Y$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $g \circ f$ is almost contra pre-open.

**Proof:** (a) Let A be an r-open set in $X$. Then $f(A)$ is $g$-open set in $Y \Rightarrow f(A)$ is open in $Y$ as $Y$ is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is pre-closed in $Z$ since $g$ is contra pre-open. Hence $gof$ is almost contra pre-open.

(b) Let A be an r-open set in $X$. Then $f$(A) is $g$-open set in $Y \Rightarrow f$ (A) is r-open in $Y$ as $Y$ is $r$-$T_{1/2} \Rightarrow g(f(A))$ is $r$-closed in $Z$ since $g$ is contra $r$-open $\Rightarrow g \circ f$ (A) is pre-closed in $Z$. Hence $gof$ is almost contra pre-open.

**Theorem 3.12:** If $f:X\to Y$ is rg-open, $g:Y\to Z$ is contra pre-open and $Y$ is $r$-$T_{1/2}$, then $gof$ is almost contra pre-open.
**Proof:** Let A be an r-open set in $X$. Then $f$(A) is rg-open in $Y \Rightarrow f$(A) is $r$-open in $Y$ since $Y$ is $r$-$T_{1/2} \Rightarrow$ g($f$(A)) = $gof$(A) is pre-closed in $Z$. Hence $gof$ is almost contra pre-open.

**Corollary 3.4:**
(i) If $f:X\to Y$ is $g$-open, $g:Y\to Z$ is contra-open[contra $r$-open] and $Y$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $gof$ is almost contra pre-open.
(ii) If $f:X\to Y$ is rg-open, g:$Y\to Z$ is contra pre-open [contra $r$-open] and $Y$ is $r$-$T_{1/2}$, then $gof$ is almost contra pre-open.

**Theorem 3.13:** If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is contra pre-open then the following statements are true.
a) If $f$ is continuous [$r$-continuous] and surjective then $g$ is almost contra pre-open.
b) If $f$ is g-continuous, surjective and $X$ is $T_{1/2}$ then $g$ is almost contra pre-open.
c) If $f$ is rg-continuous, surjective and $X$ is $r$-$T_{1/2}$ then $g$ is almost contra pre-open.
**Proof:** (a) Let A be an r-open set in $Y \Rightarrow f^{-1}$(A) is open in $X \Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ is pre-closed in $Z$. Hence $g$ is almost contra pre-open.

(b) Let A be an r-open set in $Y \Rightarrow f^{-1}$(A) is g-open in $X \Rightarrow f^{-1}$(A) is open in $X$[since $X$ is $T_{1/2}$] $\Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ is pre-closed in $Z$. Hence $g$ is almost contra pre-open.

(c) Let A be an r-open set in $Y \Rightarrow f^{-1}$(A) is rg-open in $X \Rightarrow f^{-1}$(A) is r-open in $X$[since $X$ is $r$-$T_{1/2}$] $\Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ is pre-closed in $Z$. Hence $g$ is almost contra pre-open.

**Corollary 3.5:** If $f:X\to Y$, g:$Y\to Z$ be two mappings such that $gof$ is contra-open [contra $r$-open] then the following statements are true.
a) If $f$ is continuous [$r$-continuous] and surjective then $g$ is almost contra pre-open.
b) If $f$ is g continuous, surjective and $X$ is $T_{1/2}$ then $g$ is almost contra pre-open.
c) If $f$ is rg-continuous, surjective and $X$ is $r$-$T_{1/2}$ then $g$ is almost contra pre-open.

**Theorem 3.14:** If $f:X\to Y$ is almost contra pre-open and A is an open set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-open.
**Proof:** (a) Let F be an r-open set in $A$. Then F = A$\cap$E for some open set E of $X$ and so F is open in $X \Rightarrow f$(F) is pre-closed in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is almost contra pre-open.

**Theorem 3.15:** If $f:X\to Y$ is almost contra pre-open, $X$ is $T_{1/2}$ and A is g-open set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-open.
**Proof:** Let F be an r-open set in $A$. Then F = A$\cap$E for some open set E of $X$ and so F is open in $X \Rightarrow f$(F) is pre-closed in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is almost contra pre-open.

**Corollary 3.6:** If $f:X\to Y$ is almost contra-open[almost contra $r$-open]
(i) and A is an open set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-open.
(ii) $X$ is $T_{1/2}$ and A is g-open set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-open.

**Theorem 3.16:** If $f_i$: $X_i\to Y_i$ be almost contra pre-open for i =1, 2. Let $f$: $X_1$x$X_2\to Y_1$x$Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$. Then $f:X_1$x$X_2\to Y_1$x$Y_2$ is almost contra pre-open.
**Proof:** Let $U_1$x$U_2 \subseteq X_1$x$X_2$ where $U_i$ is r-open in $X_i$ for i = 1,2. Then $f(U_1$x$U_2) = f_1(U_1)$ x $f_2(U_2)$ is pre-closed set in $Y_1$x$Y_2$. Then $f(U_1$x$U_2)$ is pre-closed set in $Y_1$x$Y_2$. Hence $f$ is almost contra pre-open.

**Corollary 3.7:** If $f_i$: $X_i\to Y_i$ be almost contra-open [almost contra $r$-open] for i =1,2. Let $f:X_1$x$X_2\to Y_1$x$Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$, then $f:X_1$x$X_2\to Y_1$x$Y_2$ is almost contra pre-open.

**Theorem 3.17:** Let $h:X\to X_1 x X_2$ be almost contra pre-open. Let $f_i:X\to X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then $f_i: X\to X_i$ is almost contra pre-open for i =1, 2.
**Proof:** Let $U_1$ be r-open in $X_1$, then $U_1 x\ X_2$ is r-open in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is pre-closed in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore $f_1$ is almost contra pre-open. Similarly we can show that $f_2$ is also almost contra pre-open and thus $f_i$: $X \to X_i$ is almost contra pre-open for i = 1, 2.

**Corollary 3.8:** Let $h$: $X\to X_1 x X_2$ be almost contra-open [almost contra *r*-open]. Let $f_i$: $X\to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i$: $X\to X_i$ is almost contra pre-open for i =1,2.

## Almost Contra Pre-Closed Mappings

**Definition 4.1:** A function $f: X\to Y$ is said to be almost contra pre-closed if the image of every r-closed set in $X$ is pre-open in $Y$.

**Example 6:** Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, X\}$; $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f:X\to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is almost contra-pre-closed and contra-pre-closed.

**Example 7:** Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Assume $f: X\to Y$ be the identity map. Then $f$ is not almost contra-pre-closed.

**Note 2:** We have the following implication diagram among the closed maps.

$$\begin{array}{ccccccc}
 & & \text{c.s.c.} & \to\ \to\to\to\to\to & & & \text{c.}\beta\text{.c} \\
 & & \downarrow & & & & \downarrow \\
 & & \text{al.c.s.c.} & \to\ \to\ \to\to\to & & & \text{al.c.}\beta\text{.c} \\
 & & \uparrow & & & & \uparrow \\
\text{Al.c.r.c} & \to\to\to & \text{al.c.c.} & \to & \text{al.c.}\alpha\text{.c.} & \to & \text{al.c.p.c.} \\
 & & \uparrow & & \uparrow & & \uparrow \\
\text{c.r.c} & \to\to\to\to & \text{c.c.} & \to\to & \text{c.}\alpha\text{.c.} & \to\to & \text{c.p.c.}
\end{array}$$

None is reversible.

**Theorem 4.1:** Every contra-pre-closed map is almost contra-pre-closed but not conversely.
**Proof:** Let $A\subseteq X$ be r-closed $\Rightarrow$ A is closed $\Rightarrow$ $f(A)$ is pre-open in $Y$. since $f:X\to Y$ is contra-pre-closed. Hence $f$ is almost contra-pre-closed.

**Example 8:** Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, X\}$; $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f:X\to Y$ be defined $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then $f$ is almost contra-pre-closed but not contra-pre-closed.

**Theorem 4.2:**
(i) If $RO(Y) = PO(Y)$ then $f$ is almost contra r-closed iff $f$ is almost contra pre-closed.
(ii) If $PO(Y) = RO(Y)$ then $f$ is almost contra-closed iff $f$ is almost contra pre-closed.
(iii)If $PO(Y) = \alpha O(Y)$ then $f$ is almost contra $\alpha$-closed iff $f$ is almost contra pre-closed.

**Proof:** Follows from note 2 and remark 1.

**Theorem 4.3:** If $f$ is closed[r-closed] and $g$ is contra pre-closed then $g$ *o* $f$ is almost contra pre-closed.

**Proof:** Let $A\subseteq X$ be r-closed$\Rightarrow$ $f(A)$ is closed[r-closed] in $Y \Rightarrow g(f(A)) = gof(A)$ is pre-open in $Z$. Hence $g$ *o* $f$ is almost contra pre-closed.
**Proof:** Let $A\subseteq X$ be r-closed in X $\Rightarrow f(A)$ is closed in Y $\Rightarrow g(f(A))$ is $r\alpha$-open in Z $\Rightarrow g(f(A)) = g{\bullet}f(A)$ is pre-open in Z Hence $g\bullet f$ is almost contra pre-closed.

**Theorem 4.4:** If $f$ is almost contra closed[almost contra-r-closed] and $g$ is pre-open then $g$ *o* $f$ is almost contra-pre-closed.

**Proof:** Let $A \subseteq X$ be r-closed in X $\Rightarrow$ $f$(A) is open in Y $\Rightarrow$ $g(f(A)) = g \bullet f(A)$ is pre-open in Z. Hence $g \bullet f$ is almost contra pre-closed.

**Corollary 4.1:**
(i) If $f$ is closed[r-closed] and $g$ is contra-closed[contra $r$-closed] then $gof$ is almost contra pre-closed.
(ii) If $f$ is almost contra closed[almost contra-r-closed] and $g$ is open[r-open] then $g \ o \ f$ is almost contra pre-closed.

**Theorem 4.5:** If $f: X \to Y$ is almost contra pre-closed, then $f(A^o) \subset p(f(A))^o$

**Proof:** Let $A \subseteq X$ be r-closed and $f: X \to Y$ is almost contra pre-closed gives $f(A^o)$ is pre-open in Y and $f(A^o) \subset f(A)$ which in turn gives $p(f(A^o))^o \subset p(f(A))^o$ - - - (1)
Since $f(A^o)$ is pre-open in Y, $p(f(A^o))^o = f(A^o)$ - - - - - - - - - - - - - - - - - -(2)
combining (1) and (2) we have $f(A^o) \subset p(f(A))^o$ for every subset A of X.

**Remark 4:** Converse is not true in general as shown by the following example.

**Example 9:** Let $X = Y = \{a, b, c\}$ $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X \to Y$ be the identity map. Then $f(A^o) \subset p(f(A))^o$ for every subset A of $X$. But $f$ is not contra-pre-closed since $f(\{c\}) = \{c\}$ is not pre-open.

**Example 10:** Let $X = Y = \{a, b, c\}$ $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X \to Y$ be the identity map. Then $f(A^o) \subset p(f(A))^o$ for every subset A of $X$. But $f$ is not almost contra-pre-closed since $f(\{a, c\}) = \{a, c\}$ and $f(\{b, c\}) = \{b, c\}$ are not pre-open.

**Corollary 4.2:** If $f: X \to Y$ is almost contra $r$-closed, then $f(A^o) \subset p(f(A))^o$

**Proof:** Let $A \subseteq X$ be r-closed and $f: X \to Y$ is almost contra $r$-closed gives $f(A^o)$ is $r$-open in Y and $f(A^o) \subset f(A)$ which in turn gives $p(f(A^o))^o \subset p(f(A))^o$ - - - - - - - - - - -(1)
Since $f(A^o)$ is pre-open in Y, $p(f(A^o))^o = f(A^o)$ - - - - - - - - - - - - - - - - - - - - - - - - -(2)
combining (1) and (2) we have $f(A^o) \subset p(f(A))^o$ for every subset A of X.

**Theorem 4.6:** If $f: X \to Y$ is almost contra pre-closed and $A \subseteq X$ is r-closed, $f$(A) is $\tau_{pre}$-open in $Y$.

***Proof:*** Let $A \subset X$ be r-closed and $f: X \to Y$ is almost contra pre-closed $\Rightarrow f(A^o) \subset p(f(A))^o \Rightarrow f(A) \subset p(f(A))^o$, since $f(A) = f(A^o)$. But $p(f(A))^o \subset f(A)$. Combining we get $f(A) = p(f(A))^o$. Therefore $f$(A) is $\tau_{pre}$-open in Y.

**Corollary 4.3:** If $f: X \to Y$ is almost contra $r$-closed, then $f$(A) is $\tau_{pre}$-open in $Y$ if A is $r$-closed set in $X$.

***Proof:*** Let $A \subset X$ be r-closed and $f: X \to Y$ is almost contra $r$-closed $\Rightarrow f(A^o) \subset r(f(A))^o \Rightarrow f(A^o) \subset p(f(A))^o$ (by theorem 4.5) $\Rightarrow f(A) \subset p(f(A))^o$, since $f(A) = f(A^o)$. But $p(f(A))^o \subset f(A)$. Combining we get $f(A) = pre(f(A))^o$. Hence $f$(A) is $\tau_{pre}$-open in $Y$.

**Theorem 4.7:** $f: X \to Y$ is almost contra pre-closed iff for each subset S of $Y$ and each r-open set U containing $f^{-1}(S)$, there is an pre-closed set V of $Y$ such that $S \subseteq V$ and $f^{1}(V) \subseteq U$.

**Remark 5:** Composition of two almost contra pre-closed maps is not almost contra pre-closed in general.

**Theorem 4.8:** Let $X$, $Y$, $Z$ be topological spaces and every pre-open set is $r$-closed in $Y$. Then the composition of two almost contra pre-closed[almost contra $r$-closed] maps is almost contra pre-closed.

**Proof:** (a) Let $f: X \to Y$ and $g: Y \to Z$ be almost contra pre-closed maps. Let A be any r-closed set in $X \Rightarrow f$ (A) is pre-open in $Y \Rightarrow f$ (A) is r-closed in $Y$ (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is pre-open in $Z$. Therefore $g \ o \ f$ is almost contra pre-closed.

(b) Let $f:X\to Y$ and $g:Y\to Z$ be almost contra pre-closed maps. Let A be any r-closed set in $X \Rightarrow f(A)$ is $r$-open in $Y \Rightarrow f(A)$ is pre-open in $Y \Rightarrow f(A)$ is r-closed in $Y$ (by assumption) $\Rightarrow g(f(A))$ is $r$-open in $Z \Rightarrow gof(A)$ is pre-open in $Z$. Therefore $gof$ is almost contra pre-closed.

**Theorem 4.9:** Let $X$, $Y$, $Z$ be topological spaces and Y is discrete topological space in $Y$. Then the composition of two almost contra pre-closed[almost contra $r$-closed] maps is almost contra pre-closed.

**Theorem 4.10:** If $f:X\to Y$ is g-closed, $g:Y\to Z$ is contra pre-closed and $Y$ is $T_{1/2}$ then $gof$ is almost contra pre-closed.

**Proof:** Let A be r-closed set in $X$. Then $f(A)$ is $g$-closed set in $Y \Rightarrow f(A)$ is closed in $Y$ as $Y$ is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is pre-open in $Z$ since $g$ is contra pre-closed. Hence $gof$ is almost contra pre-closed.

**Theorem 4.11:** If $f:X\to Y$ is rg-closed, $g:Y\to Z$ is contra pre-closed and $Y$ is $r$-$T_{1/2}$, then $g\ o\ f$ is almost contra pre-closed.

**Proof:** Let A be r-closed set in $X$. Then $f(A)$ is rg-closed in $Y \Rightarrow f(A)$ is $r$-closed in $Y$ since $Y$ is $r$-$T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is pre-open in $Z$. Hence $gof$ is almost contra pre-closed.

**Corollary 4.4:**
(i) If $f:X\to Y$ is $g$-open, $g:Y\to Z$ is contra-closed [contra $r$-closed] and $Y$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $gof$ is almost contra pre-closed.
(ii)If $f:X\to Y$ is rg-open, $g:Y\to Z$ is contra-closed [contra $r$-closed] and $Y$ is $r$-$T_{1/2}$, then $g\ o\ f$ is almost contra pre-closed.

**Theorem 4.12:** If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is contra pre-closed then the following statements are true.
i) If $f$ is continuous [$r$-continuous] and surjective then $g$ is almost contra pre-closed.
ii) If $f$ is g-continuous, surjective and $X$ is $T_{1/2}$ then $g$ is almost contra pre-closed.
iii) If $f$ is rg-continuous, surjective and $X$ is $r$-$T_{1/2}$ then $g$ is almost contra pre-closed.

**Proof:** (a) Let A be r-closed set in $Y \Rightarrow f^{-1}(A)$ is closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ is pre-open in $Z$. Hence $g$ is almost contra pre-closed.

(b) Let A be r-closed set in $Y. \Rightarrow f^{-1}(A)$ is g-closed in $X \Rightarrow f^{-1}(A)$ is closed in $X$[since $X$ is $T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ is pre-open in $Z$. Hence $g$ is almost contra pre-closed.

(c) Let A be r-closed set in $Y \Rightarrow f^{-1}(A)$ is g-closed in $X \Rightarrow f^{-1}(A)$ is closed in $X$[since $X$ is $r$-$T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ is pre-open in $Z$. Hence $g$ is almost contra pre-closed.

**Corollary 4.5:** If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is contra-closed [contra $r$-closed] then the following statements are true.
i) If $f$ is continuous [$r$-continuous] and surjective then $g$ is almost contra pre-closed.
ii) If $f$ is g continuous, surjective and $X$ is $T_{1/2}$ then $g$ is almost contra pre-closed.
iii) If $f$ is rg-continuous, surjective and $X$ is $r$-$T_{1/2}$ then $g$ is almost contra pre-closed.

**Theorem 4.13:** If $f:X\to Y$ is almost contra pre-closed and A is an closed set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-closed.

**Proof:** (a) Let F be r-closed set in $A$. Then F = A$\cap$E for some closed set E of $X$ and so F is closed in $X \Rightarrow f(F)$ is pre-open in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is almost contra pre-closed.

**Theorem 4.14:** If $f:X\to Y$ is almost contra pre-closed, $X$ is $T_{1/2}$ and A is g-closed set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-closed.

**Proof:** Let F be r-closed set in $A$. Then F = A∩E for some closed set E of $X$ and so F is closed in $X \Rightarrow f(F)$ is pre-open in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is almost contra pre-closed.

**Corollary 4.6:** If $f:X\to Y$ is almost contra-closed [almost contra $r$-closed],
(i) and A is an closed set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-closed.
(ii) $X$ is $T_{1/2}$, A is g-closed set of $X$ then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is almost contra pre-closed.

**Theorem 4.15:** If $f_i:X_i\to Y_i$ be almost contra pre-closed for i = 1, 2. Let $f: X_1 x X_2 \to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$. Then $f:X_1 x X_2\to Y_1 x Y_2$ is almost contra pre-closed.

**Proof:** Let $U_1 x U_2 \subseteq X_1 x X_2$ where $U_i$ is r-closed in $X_i$ for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is pre-open set in $Y_1 x Y_2$. Then $f(U_1 x U_2)$ is pre-open set in $Y_1 x Y_2$. Hence $f$ is almost contra pre-closed.

**Corollary 4.7:** If $f_i$: $X_i\to Y_i$ be almost contra-closed [almost contra $r$-closed] for i =1,2. Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is almost contra pre-closed.

**Theorem 4.16:** Let $h:X\to X_1 x X_2$ be almost contra pre-closed. Let $f_i:X\to X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then $f_i$: $X\to X_i$ is almost contra pre-closed for i =1, 2.

**Proof:** Let $U_1$ be r-closed in $X_1$, then $U_1 x\ X_2$ is r-closed in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is pre-open in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore $f_1$ is almost contra pre-closed. Similarly we can show that $f_2$ is also almost contra pre-closed and thus $f_i$: X $\to X_i$ is almost contra pre-closed for i = 1, 2.

**Corollary 4.8:** Let $h$: $X\to X_1 x X_2$ be almost contra-closed. Let $f_i$: $X\to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i$: $X\to X_i$ is almost contra pre-closed for i =1,2.

## Conclusion

In this paper we introduced the concept of almost contra pre-open and almost contra-pre closed mappings, studied their basic properties and the interrelationship between other open and closed maps.

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